AIAA JOURNAL
Vol. 34, No. 11, November 1996

Initial Fatigue Quality Confidence-Interval Approach for Determination of Inspection Intervals

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An approach is proposed for the determination of the inspection interval of metallic aerospace components; it makes use of the Initial Fatigue Quality Model proposed by the U.S. Air Force. Once the structure has been qualified through the latter model, an upper and a lower bound for the inspection time are obtained. Such bounds are derived on the basis of a statistical procedure based on the knowledge of confidence intervals. Tables of general applicability have been obtained through Monte Carlo simulations to allow the determination of the interval in which the inspection time should lie. An example problem is discussed to show an application of the proposed method.

Nomenclature

a_{P_1} = percentile of equivalent initial flaw size (EIFS) distribution corresponding to P_1
a_{P_2} = percentile of EIFS distribution corresponding to P_2
$a_{(0)}$ = equivalent initial flaw size, i.e., crack length at time zero
a_0 = reference crack length at time t_0
a_1 = lower bound of crack-length dimension with relation to the inspection-interval identification
a_2 = upper bound of crack-length dimension with relation to the inspection-interval identification
b = parameter of crack growth law
$F_{a(0)}$ = cumulative probability function of EIFS, equal to P_2
$\hat{F}_{a(0)}$ = estimated cumulative probability function of EIFS
F_T = cumulative probability function of time to crack initiation (TTCI)
N = number of elements in a data sample
$P[a \leq x]$ = cumulative probability function of EIFS
$P[t \leq T]$ = cumulative probability function of TTCI
P_1 = probability of having a crack with dimension greater than a given one
P_2 = probability of having a crack with dimension lower than a given one, equal to $F_{a(0)}$
$\hat{P}_1$ = estimated value of P_1
$\hat{P}_2$ = estimated value of P_2
$\underline{P}_{1l}$ = lower confidence bound of P_1
$\underline{P}_{2l}$ = lower confidence bound of P_2
$\bar{P}_{2u}$ = upper confidence bound of P_2
Q = parameter of crack growth law
T = time to crack initiation
t = service time
x_u = upper bound of EIFS probability distribution function
α = shape parameter of Weibull probability distribution function
$\hat{\alpha}$ = estimated shape parameter of Weibull probability distribution function
β = scale parameter of Weibull probability distribution function
$\hat{\beta}$ = estimated scale parameter of Weibull probability distribution function
ε = lower bound of Weibull probability distribution function
K = parameter related to uniform distributed data set in the Monte Carlo simulation by using the maximum likelihood method
K′ = parameter related to uniform distributed data set in the Monte Carlo simulation by using the least-squares method
λ = parameter related to uniform distributed data set in the Monte Carlo simulation by using the maximum likelihood method
λ' = parameter related to uniform distributed data set in the Monte Carlo simulation by using the least-squares method
$\hat{\phi}$ = estimated scale parameter of EIFS cumulative probability distribution function

Introduction

A PROBABILISTIC approach to durability analysis of aerospace structures has been proposed by the U.S. Air Force (USAF).[1–9] According to this approach, the distribution of the damage in a structure is studied by considering the population of cracks that nucleate from stress concentration zones in the analyzed component, for instance, fastener holes, cutouts. The crack length is modeled as a random variable. The user can monitor the evolution of the damage in the structure, through the estimation of the random variable defined above, and therefore evaluate quantitatively how the capability of the structure for carrying load is degraded; through this knowledge, one can decide the location in time for inspection and upgrading intervals. Using the USAF approach, one obtains the service time corresponding to a fixed probability of crack exceedance for a choosen crack length. Consequently, it is possible to estimate the number of cracks in the structure with dimension greater than a reference one and a corresponding γ-level confidence interval. The approach proposed herein follows the USAF methodology for estimating the probability of crack exceedance, but gives as output a time interval such that for a service time belonging to such interval, there is a certain probability, say γ, that the probability of exceeding a fixed crack length, say a_1, is greater than a fixed value, say P_1, and that the probability of having a crack length lower than a fixed dimension, say a_2, is greater than a fixed value, say P_2. In this way, the inspection will take place at a time where the economic benefit is greater, because the structure is damaged to some extent, but the degradation is still limited to allow economical upgrading of the structure. The USAF methodology is summarized, and the proposed approach is discussed in detail.

USAF Initial Fatigue Quality Method

According to the USAF method, the dimension of the crack is modeled as a random variable. Each flaw, because of the limited range of length considered in comparison to the distance between the

Received Dec. 18, 1995; revision received May 15, 1996; accepted for publication June 6, 1996; also published in *AIAA Journal on Disc*, Volume 1, Number 4.

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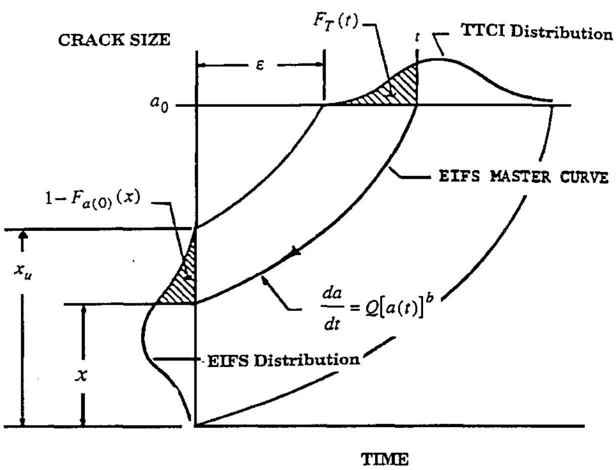

Fig. 1 Initial Fatigue Quality Model.[8]

stress concentration zones where the cracks originate, is independent of the others originating from nearby locations; i.e., it is assumed that the situation studied is such that no interaction takes place. Once the probability distribution function of having a flaw with a dimension greater than a fixed value has been estimated, an assessment of the damage to the structure can be obtained. In fact, at a given time, the probability of crack exceedance is known and, under the assumption of binomial distribution, an estimation of the number of cracks longer than a reference value is easily calculated. The inspection interval should be planned in such a way that, when the number of cracks with dimension greater than a reference value exceeds a fixed value, the aircraft must be grounded and the structure upgraded. In the case of riveted components, the rivets must be removed, the holes reamed and larger-diameter rivets installed, thus restoring the fatigue quality level of the structure. The approach proposed by the USAF makes use of two statistical distributions,[8] namely, 1) the distribution of the time to crack initiation (TTCI) and 2) the distribution of the equivalent initial flow size (EIFS) (see Fig. 1). The TTCI distribution represents the time necessary to reach a fixed length for the largest crack in the stress concentration zone. Once the TTCI distribution has been evaluated, one can obtain the EIFS distribution by backextrapolating the crack length to time zero, as explained below. The TTCI is described by means of a three-parameter Weibull distribution as follows:

$$F_T(t) \equiv P[T \leq t] = 1 - \exp\{-[(t - \varepsilon)/\beta]^\alpha\} \tag{1}$$

where $F_T(t)$gives the probability that the crack has reached the reference crack length at a service time T smaller than t, whereas its complementary value, i.e.,

$$1 - F_T(t) = P[T > t] = \exp\{-[(t - \varepsilon)/\beta]^\alpha\} \tag{2}$$

obviously represents the probability that the crack reaches the reference crack length at a time T greater than t.

The EIFS distribution is obtained from the TTCI distribution by using a backextrapolation method.[8] The following expression holds:

$$F_T(t) = 1 - F_{a(0)}(x) \tag{3}$$

where

$$F_{a(0)}(x) \equiv P[a \leq x] \tag{4}$$

and

$$\frac{\mathrm{d}a}{\mathrm{d}t} = Qa^b \tag{5}$$

where a is the crack length and Q and b are parameters that depend on the component material and geometry and on the load history. In the present analysis, it is assumed that a specific situation is studied so that Q and b can be considered constant; further it is assumed that $b = 1$, as in some cases examined by Manning and Yang.[8] Considering Eqs. (1), (3), and (5), one obtains

$$\begin{aligned} F_{a(0)} &\equiv \exp\left\{-\left[\frac{\ln(x_u/x)}{\phi}\right]^\alpha\right\}; \qquad 0 \leq x \leq x_u \\ &\equiv 1.0; \qquad x \geq x_u \end{aligned} \tag{6}$$

where x_u is the EIFS upper-bound limit, corresponding to the TTCI lower-bound limit, and ϕ is related through Eq. (5) to the parameters of the TTCI distribution function. Indeed, the following relationship holds:

$$x_u = a_0 \exp(-Q\varepsilon) \tag{7}$$

$$\phi = Q\beta \tag{8}$$

The EIFS probability function also is called Weibull compatible distribution because of Eq. (3) (Ref. 8).

Once the EIFS distribution function is known, one can obtain the probability of crack exceedance of a given crack length at any time using Eqs. (5) and (6) (Ref. 8).

Here, it is assumed that the lower bound of the distribution of the TTCI, ε, is either known or equal to zero, this hypothesis being equivalent to assume that the probability of having a crack equal to the reference crack length may be very low but nonzero at a time immediately following the airplane going into service. Accordingly, Eq. (1) becomes

$$F_T(t) \equiv P[T \leq t] = 1 - \exp[-(t/\beta)^\alpha] \tag{9}$$

whereas Eq. (6) is now

$$\begin{aligned} F_{a(0)} &\equiv \exp\left\{-\left[\frac{\ln(a_0/x)}{\phi}\right]^\alpha\right\}; \qquad 0 \leq x \leq a_0 \\ &\equiv 1.0; \qquad x > a_0 \end{aligned} \tag{10}$$

Proposed Methodology

The choice of the upgrading time can be influenced by two different factors. The first one is attributable to the consideration that one may be interested in upgrading the structure once a certain number of cracks is present in it, so that the upgrading is useful and gives economic benefits. In the second place, one is interested in executing such an action before a fixed number of cracks has reached a length for which the repair would lead to a higher cost; for example, the crack starting from a rivet hole is too long to allow the reaming of the hole, hence the substitution of the component is necessary. Suppose one is interested in reaming the component holes in a service time belonging to the interval $[t_1, t_2]$, where t_1 corresponds to the time such that the probability of having a crack longer than a_1 is P_1, and t_2 to the time such that the probability of having a crack of length smaller than a_2 is P_2. Given the values of P_1 and P_2, it will be shown that it is possible to obtain the values of the corresponding service times, t_1 and t_2. Once the EIFS probability distribution function is known, one can obtain the percentile of such a distribution corresponding to P_1 and P_2, being a_{P_1} and a_{P_2}, respectively. This is done by substituting P_2 and $1 - P_1$ for $F_{a(0)}$ in Eq. (6) and obtaining x, i.e., a_{P_1} and a_{P_2}. By integrating Eq. (5), the time interval $[t_1, t_2]$ is obtained in which the inspection time should lie. In fact, it is

$$\ln a_1 - \ln a_{P_1} = Qt_1 \tag{11}$$

$$\ln a_2 - \ln a_{P_2} = Qt_2 \tag{12}$$

The determination of the probability of crack exceedance is strongly affected by the quality of the estimation of the values of the parameters of the Weibull probability distribution function. To take into account the fact that one does not know the actual value of the parameters but only their estimates, one should consider that, given a value of $F_{a(0)}$, x is a random variable and vice versa. If it is possible to obtain a γ confidence interval for P_1 and P_2, then one

Table 1 Lower confidence bound of EIFS distribution (γ = 0.90), ML parameter estimation

	N																			
$\bar{P}_{2l}$	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.50	0.273	0.336	0.365	0.383	0.395	0.404	0.411	0.417	0.421	0.425	0.429	0.432	0.435	0.437	0.440	0.441	0.443	0.444	0.446	0.447
0.52	0.286	0.353	0.383	0.401	0.413	0.423	0.430	0.436	0.440	0.444	0.448	0.451	0.454	0.457	0.459	0.461	0.463	0.464	0.466	0.467
0.54	0.300	0.370	0.400	0.420	0.432	0.442	0.449	0.455	0.459	0.464	0.468	0.471	0.474	0.476	0.479	0.480	0.482	0.483	0.485	0.487
0.56	0.314	0.387	0.418	0.438	0.451	0.460	0.468	0.475	0.479	0.484	0.487	0.490	0.494	0.496	0.498	0.500	0.502	0.503	0.505	0.506
0.58	0.328	0.404	0.437	0.457	0.470	0.480	0.488	0.494	0.498	0.503	0.507	0.510	0.513	0.516	0.518	0.520	0.522	0.523	0.525	0.526
0.60	0.343	0.421	0.455	0.476	0.489	0.499	0.507	0.514	0.518	0.523	0.527	0.530	0.533	0.535	0.538	0.540	0.542	0.543	0.545	0.546
0.62	0.358	0.439	0.474	0.495	0.508	0.518	0.527	0.533	0.538	0.543	0.547	0.550	0.553	0.555	0.558	0.560	0.562	0.563	0.565	0.566
0.64	0.373	0.457	0.493	0.515	0.528	0.538	0.546	0.553	0.558	0.563	0.567	0.570	0.573	0.576	0.578	0.580	0.582	0.583	0.585	0.587
0.66	0.388	0.475	0.513	0.534	0.548	0.558	0.566	0.573	0.578	0.583	0.587	0.590	0.593	0.596	0.598	0.600	0.602	0.603	0.605	0.607
0.68	0.404	0.494	0.532	0.554	0.568	0.579	0.587	0.594	0.598	0.604	0.607	0.611	0.614	0.616	0.619	0.621	0.623	0.624	0.626	0.627
0.70	0.420	0.513	0.552	0.575	0.589	0.599	0.607	0.614	0.619	0.624	0.628	0.631	0.634	0.637	0.639	0.641	0.643	0.645	0.646	0.648
0.72	0.437	0.533	0.572	0.595	0.610	0.620	0.628	0.635	0.640	0.645	0.649	0.652	0.655	0.658	0.660	0.662	0.664	0.665	0.667	0.669
0.74	0.454	0.554	0.593	0.616	0.631	0.641	0.649	0.656	0.661	0.666	0.670	0.673	0.676	0.679	0.681	0.683	0.685	0.687	0.688	0.690
0.76	0.472	0.574	0.614	0.638	0.652	0.663	0.671	0.678	0.682	0.688	0.691	0.694	0.698	0.700	0.702	0.704	0.706	0.708	0.709	0.711
0.78	0.490	0.596	0.636	0.660	0.674	0.685	0.693	0.700	0.704	0.709	0.713	0.716	0.719	0.722	0.724	0.726	0.727	0.729	0.730	0.732
0.80	0.510	0.618	0.659	0.682	0.696	0.707	0.715	0.722	0.726	0.732	0.735	0.738	0.741	0.743	0.746	0.748	0.749	0.751	0.752	0.754
0.82	0.531	0.641	0.682	0.706	0.720	0.730	0.738	0.745	0.749	0.754	0.757	0.760	0.763	0.766	0.768	0.770	0.771	0.773	0.774	0.775
0.84	0.553	0.665	0.706	0.729	0.743	0.754	0.762	0.768	0.772	0.777	0.780	0.783	0.786	0.788	0.790	0.792	0.793	0.795	0.796	0.798
0.86	0.577	0.691	0.731	0.754	0.768	0.778	0.786	0.792	0.795	0.800	0.804	0.806	0.809	0.811	0.813	0.815	0.816	0.818	0.819	0.820
0.88	0.603	0.718	0.758	0.780	0.793	0.803	0.810	0.816	0.820	0.824	0.827	0.830	0.833	0.834	0.836	0.838	0.839	0.841	0.842	0.843
0.90	0.632	0.747	0.786	0.807	0.820	0.829	0.836	0.841	0.845	0.849	0.852	0.854	0.857	0.859	0.860	0.862	0.863	0.864	0.865	0.866
0.92	0.663	0.779	0.816	0.836	0.848	0.856	0.863	0.867	0.871	0.875	0.877	0.879	0.882	0.883	0.885	0.886	0.887	0.888	0.889	0.890
0.94	0.701	0.814	0.849	0.867	0.878	0.885	0.891	0.895	0.898	0.901	0.904	0.906	0.907	0.909	0.910	0.911	0.912	0.913	0.914	0.915
0.96	0.746	0.855	0.885	0.901	0.910	0.916	0.921	0.925	0.927	0.930	0.932	0.933	0.934	0.936	0.937	0.938	0.938	0.939	0.940	0.940
0.98	0.808	0.904	0.929	0.940	0.947	0.951	0.955	0.957	0.959	0.961	0.962	0.963	0.964	0.965	0.965	0.966	0.966	0.967	0.967	0.968

Table 2 Lower confidence bound of EIFS distribution (γ = 0.95), ML parameter estimation

	N																			
$\bar{P}_{2l}$	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.50	0.220	0.295	0.331	0.351	0.366	0.377	0.386	0.393	0.399	0.405	0.409	0.413	0.417	0.420	0.423	0.425	0.427	0.429	0.431	0.433
0.52	0.232	0.311	0.348	0.369	0.384	0.396	0.405	0.412	0.418	0.424	0.429	0.432	0.436	0.439	0.442	0.444	0.446	0.448	0.450	0.452
0.54	0.245	0.327	0.364	0.386	0.402	0.414	0.424	0.431	0.437	0.443	0.448	0.451	0.455	0.458	0.461	0.464	0.466	0.468	0.470	0.472
0.56	0.257	0.343	0.382	0.404	0.421	0.433	0.442	0.450	0.456	0.463	0.467	0.471	0.475	0.478	0.481	0.483	0.485	0.487	0.489	0.492
0.58	0.270	0.359	0.399	0.423	0.439	0.452	0.462	0.469	0.476	0.482	0.487	0.490	0.494	0.497	0.501	0.503	0.505	0.507	0.509	0.511
0.60	0.283	0.376	0.417	0.441	0.458	0.471	0.481	0.489	0.495	0.502	0.506	0.510	0.514	0.517	0.521	0.523	0.525	0.527	0.529	0.531
0.62	0.296	0.393	0.435	0.460	0.477	0.491	0.500	0.509	0.515	0.522	0.526	0.530	0.534	0.537	0.541	0.543	0.545	0.547	0.549	0.551
0.64	0.309	0.410	0.453	0.479	0.497	0.510	0.520	0.529	0.535	0.542	0.546	0.550	0.554	0.557	0.561	0.563	0.565	0.567	0.569	0.571
0.66	0.323	0.428	0.472	0.498	0.516	0.530	0.540	0.549	0.555	0.562	0.566	0.570	0.574	0.578	0.581	0.584	0.586	0.588	0.590	0.592
0.68	0.337	0.447	0.491	0.518	0.537	0.550	0.561	0.569	0.575	0.583	0.587	0.591	0.595	0.598	0.602	0.604	0.606	0.608	0.610	0.612
0.70	0.353	0.465	0.511	0.538	0.557	0.571	0.582	0.590	0.596	0.603	0.607	0.612	0.616	0.619	0.623	0.625	0.627	0.629	0.631	0.633
0.72	0.369	0.484	0.531	0.559	0.578	0.592	0.603	0.611	0.617	0.625	0.629	0.633	0.637	0.640	0.644	0.646	0.648	0.650	0.652	0.654
0.74	0.385	0.504	0.552	0.580	0.599	0.613	0.624	0.633	0.638	0.646	0.650	0.654	0.658	0.661	0.665	0.667	0.669	0.671	0.673	0.675
0.76	0.402	0.524	0.574	0.602	0.621	0.635	0.646	0.655	0.660	0.668	0.672	0.676	0.680	0.683	0.686	0.688	0.691	0.692	0.694	0.697
0.78	0.420	0.546	0.596	0.624	0.644	0.657	0.668	0.677	0.683	0.690	0.694	0.698	0.702	0.705	0.708	0.710	0.713	0.714	0.716	0.718
0.80	0.438	0.568	0.619	0.647	0.666	0.680	0.691	0.700	0.705	0.712	0.716	0.720	0.724	0.727	0.730	0.733	0.735	0.736	0.738	0.740
0.82	0.457	0.590	0.643	0.671	0.690	0.704	0.714	0.723	0.728	0.735	0.739	0.743	0.747	0.750	0.753	0.755	0.757	0.759	0.761	0.763
0.84	0.479	0.615	0.667	0.696	0.715	0.728	0.738	0.747	0.752	0.759	0.763	0.767	0.770	0.773	0.776	0.778	0.780	0.782	0.783	0.785
0.86	0.502	0.642	0.693	0.721	0.740	0.753	0.763	0.772	0.777	0.783	0.787	0.790	0.794	0.797	0.800	0.801	0.803	0.805	0.807	0.808
0.88	0.527	0.669	0.721	0.749	0.767	0.779	0.789	0.797	0.802	0.808	0.812	0.815	0.818	0.821	0.824	0.825	0.827	0.829	0.830	0.832
0.90	0.555	0.699	0.750	0.777	0.795	0.807	0.816	0.824	0.828	0.834	0.837	0.841	0.844	0.846	0.848	0.850	0.852	0.853	0.855	0.856
0.92	0.587	0.733	0.782	0.808	0.825	0.836	0.844	0.852	0.856	0.861	0.864	0.867	0.870	0.872	0.874	0.876	0.877	0.878	0.880	0.881
0.94	0.625	0.771	0.817	0.842	0.857	0.867	0.874	0.881	0.885	0.889	0.892	0.895	0.897	0.899	0.901	0.902	0.904	0.905	0.906	0.907
0.96	0.672	0.815	0.858	0.880	0.893	0.901	0.907	0.913	0.916	0.920	0.922	0.924	0.926	0.928	0.929	0.931	0.931	0.932	0.933	0.934
0.98	0.740	0.873	0.908	0.925	0.934	0.941	0.945	0.949	0.951	0.954	0.956	0.957	0.958	0.959	0.960	0.961	0.962	0.963	0.963	0.964

could consider the lower γ confidence bound for P_1, say $\bar{P}_{1l}$, and the lower γ confidence bound for P_2, say $\bar{P}_{2l}$, thus obtaining a γ probability that the probability of having a crack with length greater than a_1 at time $\bar{t}_1$ is P_1, and a γ probability that the probability of having a crack with length smaller than a_2 at time $\bar{t}_2$, is P_2. The values of $\bar{t}_1$ and $\bar{t}_2$ are obtained from the following equations:

$$\ell n\, a_1 - \ell n\, \bar{a}_{P_1} = Q\bar{t}_1 \tag{13}$$

$$\ell n\, a_2 - \ell n\, \bar{a}_{P_2} = Q\bar{t}_2 \tag{14}$$

where $\bar{a}_{P_1}$ and $\bar{a}_{P_2}$ are the percentiles of the EIFS distribution corresponding to $\bar{P}_{1l}$ and $\bar{P}_{2l}$.

Properties of Estimated Parameters of Weibull Cumulative Probability Distribution Function

To illustrate the procedure for obtaining the confidence intervals for the distribution of the probability of crack exceedance, it is necessary to outline some properties of the parameter estimation of the Weibull probability distribution function. In the following, least-squares (LS) and maximum likelihood (ML) parameter estimation is considered.

As shown by Pieracci,[10,11] the actual Weibull distributed data set (i.e., the TTCI data set) can be thought of as if it has been obtained through a transformation from a set of uniform distributed

Table 3 Lower confidence bound of EIFS distribution (γ = 0.98), ML parameter estimation

$\bar{P}_{2l}$	N=5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.50	0.169	0.252	0.292	0.316	0.334	0.349	0.359	0.368	0.376	0.383	0.388	0.392	0.396	0.400	0.403	0.407	0.409	0.412	0.414	0.416
0.52	0.179	0.266	0.308	0.333	0.352	0.367	0.378	0.387	0.395	0.402	0.407	0.411	0.415	0.419	0.422	0.426	0.429	0.431	0.433	0.435
0.54	0.189	0.281	0.324	0.350	0.369	0.384	0.396	0.405	0.413	0.421	0.425	0.430	0.434	0.438	0.442	0.445	0.448	0.450	0.452	0.455
0.56	0.200	0.297	0.340	0.367	0.387	0.403	0.414	0.424	0.432	0.440	0.444	0.449	0.453	0.457	0.461	0.464	0.468	0.470	0.472	0.475
0.58	0.211	0.311	0.357	0.385	0.406	0.421	0.433	0.443	0.451	0.459	0.463	0.469	0.473	0.477	0.480	0.484	0.487	0.490	0.492	0.494
0.60	0.221	0.326	0.374	0.403	0.424	0.440	0.452	0.462	0.471	0.478	0.483	0.488	0.493	0.496	0.500	0.504	0.507	0.509	0.511	0.514
0.62	0.233	0.342	0.392	0.421	0.443	0.459	0.471	0.482	0.490	0.498	0.502	0.508	0.512	0.516	0.520	0.524	0.527	0.529	0.531	0.534
0.64	0.245	0.359	0.410	0.441	0.462	0.478	0.491	0.502	0.510	0.518	0.522	0.528	0.532	0.536	0.540	0.543	0.547	0.549	0.551	0.555
0.66	0.257	0.375	0.428	0.460	0.481	0.498	0.511	0.522	0.531	0.538	0.543	0.548	0.553	0.557	0.561	0.564	0.567	0.570	0.572	0.575
0.68	0.270	0.392	0.446	0.479	0.501	0.518	0.531	0.542	0.551	0.558	0.563	0.569	0.573	0.577	0.581	0.585	0.588	0.590	0.593	0.596
0.70	0.283	0.410	0.466	0.499	0.521	0.538	0.551	0.563	0.572	0.579	0.584	0.590	0.594	0.598	0.602	0.606	0.609	0.611	0.613	0.616
0.72	0.296	0.429	0.485	0.519	0.542	0.559	0.572	0.584	0.593	0.600	0.606	0.611	0.615	0.619	0.624	0.627	0.630	0.632	0.634	0.637
0.74	0.310	0.448	0.506	0.540	0.563	0.581	0.594	0.605	0.615	0.622	0.627	0.633	0.637	0.641	0.645	0.648	0.651	0.654	0.656	0.658
0.76	0.325	0.467	0.527	0.561	0.585	0.603	0.616	0.627	0.637	0.644	0.649	0.655	0.658	0.662	0.667	0.670	0.673	0.675	0.678	0.680
0.78	0.341	0.488	0.549	0.583	0.608	0.625	0.639	0.650	0.659	0.666	0.672	0.677	0.681	0.685	0.689	0.692	0.695	0.697	0.700	0.702
0.80	0.357	0.509	0.571	0.607	0.631	0.648	0.662	0.673	0.682	0.689	0.694	0.700	0.704	0.708	0.712	0.715	0.717	0.720	0.722	0.725
0.82	0.374	0.532	0.595	0.631	0.655	0.672	0.685	0.697	0.705	0.713	0.718	0.723	0.727	0.731	0.735	0.738	0.740	0.742	0.745	0.747
0.84	0.393	0.557	0.620	0.657	0.680	0.697	0.710	0.722	0.730	0.737	0.742	0.747	0.751	0.755	0.759	0.761	0.763	0.766	0.768	0.771
0.86	0.415	0.583	0.646	0.683	0.706	0.723	0.736	0.747	0.755	0.762	0.767	0.772	0.776	0.779	0.783	0.785	0.788	0.790	0.792	0.794
0.88	0.438	0.610	0.674	0.711	0.734	0.751	0.763	0.774	0.781	0.788	0.793	0.797	0.801	0.804	0.809	0.810	0.812	0.815	0.817	0.819
0.90	0.465	0.640	0.705	0.741	0.763	0.779	0.792	0.802	0.809	0.815	0.819	0.824	0.827	0.831	0.834	0.836	0.838	0.840	0.842	0.844
0.92	0.494	0.674	0.739	0.774	0.795	0.811	0.822	0.831	0.838	0.844	0.848	0.852	0.855	0.858	0.861	0.863	0.865	0.867	0.869	0.870
0.94	0.529	0.714	0.778	0.811	0.830	0.844	0.855	0.863	0.868	0.874	0.878	0.881	0.884	0.887	0.890	0.891	0.893	0.895	0.896	0.898
0.96	0.576	0.762	0.822	0.853	0.869	0.882	0.891	0.898	0.902	0.908	0.910	0.913	0.916	0.918	0.920	0.922	0.923	0.924	0.925	0.927
0.98	0.645	0.827	0.879	0.904	0.917	0.927	0.933	0.939	0.942	0.945	0.947	0.949	0.951	0.953	0.954	0.955	0.956	0.957	0.958	0.959

Table 4 Lower confidence bound of EIFS distribution (γ = 0.90), LS parameter estimation

$\bar{P}_{2l}$	N=5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.50	0.288	0.347	0.373	0.389	0.400	0.408	0.415	0.420	0.424	0.428	0.431	0.434	0.437	0.439	0.441	0.443	0.445	0.446	0.447	0.449
0.52	0.310	0.369	0.395	0.412	0.422	0.430	0.437	0.442	0.445	0.449	0.453	0.455	0.458	0.460	0.462	0.464	0.466	0.467	0.468	0.470
0.54	0.332	0.392	0.417	0.434	0.444	0.452	0.458	0.463	0.466	0.471	0.474	0.477	0.479	0.481	0.483	0.485	0.487	0.488	0.489	0.490
0.56	0.354	0.414	0.439	0.455	0.465	0.473	0.479	0.485	0.488	0.492	0.495	0.498	0.501	0.502	0.504	0.506	0.508	0.509	0.510	0.511
0.58	0.376	0.437	0.462	0.477	0.487	0.495	0.501	0.506	0.509	0.514	0.516	0.519	0.522	0.524	0.526	0.527	0.528	0.529	0.531	0.532
0.60	0.398	0.460	0.484	0.499	0.509	0.516	0.523	0.527	0.531	0.535	0.538	0.540	0.543	0.545	0.546	0.548	0.549	0.551	0.552	0.553
0.62	0.420	0.482	0.506	0.521	0.531	0.538	0.544	0.549	0.552	0.556	0.559	0.561	0.564	0.566	0.568	0.569	0.570	0.572	0.573	0.574
0.64	0.442	0.504	0.529	0.543	0.552	0.560	0.566	0.570	0.573	0.578	0.580	0.583	0.585	0.587	0.589	0.590	0.591	0.593	0.594	0.595
0.66	0.465	0.527	0.551	0.566	0.574	0.582	0.588	0.592	0.595	0.599	0.602	0.604	0.606	0.608	0.610	0.611	0.612	0.614	0.615	0.616
0.68	0.487	0.550	0.573	0.588	0.597	0.604	0.609	0.614	0.617	0.620	0.623	0.625	0.628	0.629	0.631	0.633	0.633	0.635	0.636	0.637
0.70	0.510	0.573	0.596	0.610	0.619	0.626	0.631	0.635	0.639	0.642	0.645	0.647	0.649	0.651	0.652	0.654	0.655	0.656	0.657	0.658
0.72	0.533	0.597	0.619	0.633	0.641	0.648	0.653	0.658	0.660	0.664	0.666	0.668	0.670	0.672	0.674	0.675	0.676	0.677	0.678	0.679
0.74	0.557	0.620	0.642	0.655	0.664	0.670	0.675	0.680	0.682	0.686	0.688	0.690	0.692	0.694	0.695	0.696	0.697	0.699	0.699	0.701
0.76	0.581	0.644	0.666	0.678	0.686	0.693	0.698	0.702	0.704	0.707	0.710	0.712	0.714	0.715	0.717	0.718	0.719	0.720	0.721	0.722
0.78	0.605	0.668	0.689	0.701	0.709	0.715	0.720	0.724	0.727	0.730	0.732	0.734	0.735	0.737	0.738	0.739	0.741	0.742	0.742	0.744
0.80	0.631	0.693	0.713	0.725	0.732	0.738	0.743	0.746	0.749	0.752	0.754	0.756	0.757	0.759	0.760	0.761	0.762	0.763	0.764	0.765
0.82	0.656	0.718	0.737	0.749	0.755	0.762	0.765	0.769	0.772	0.774	0.776	0.778	0.779	0.781	0.782	0.783	0.784	0.785	0.786	0.787
0.84	0.682	0.743	0.762	0.773	0.779	0.785	0.788	0.792	0.794	0.797	0.799	0.800	0.802	0.803	0.804	0.805	0.806	0.807	0.808	0.809
0.86	0.709	0.770	0.787	0.797	0.803	0.809	0.812	0.815	0.817	0.820	0.822	0.823	0.824	0.826	0.827	0.828	0.829	0.829	0.830	0.831
0.88	0.738	0.796	0.813	0.822	0.828	0.833	0.836	0.839	0.841	0.843	0.845	0.846	0.847	0.849	0.850	0.850	0.851	0.852	0.852	0.853
0.90	0.769	0.824	0.840	0.848	0.853	0.858	0.860	0.863	0.865	0.867	0.868	0.870	0.871	0.872	0.873	0.873	0.874	0.875	0.875	0.876
0.92	0.801	0.853	0.867	0.874	0.879	0.883	0.885	0.888	0.889	0.891	0.892	0.893	0.894	0.895	0.896	0.897	0.897	0.898	0.898	0.899
0.94	0.836	0.884	0.896	0.902	0.906	0.909	0.911	0.913	0.914	0.916	0.917	0.918	0.918	0.919	0.920	0.921	0.921	0.922	0.922	0.923
0.96	0.876	0.916	0.926	0.930	0.934	0.936	0.938	0.939	0.940	0.942	0.942	0.943	0.944	0.944	0.945	0.945	0.946	0.946	0.946	0.947
0.98	0.923	0.952	0.958	0.962	0.964	0.965	0.966	0.967	0.968	0.969	0.969	0.969	0.970	0.970	0.971	0.971	0.971	0.971	0.971	0.972

data. Accordingly, the LS estimated values of the parameters can be written in the following way:

$$\hat{\alpha} = K\alpha \tag{15}$$

$$\ln\hat{\beta} = \ln\beta + (\lambda/\alpha) \tag{16}$$

where the symbols with the circumflexes represent the estimated values of the parameters and the constants λ and K are only dependent on the transformation from the uniform distributed data set to the Weibull distributed data set.

Following Thoman et al.[12] and Pieracci,[10,11] for ML the estimated values of the parameters can be written as

$$\hat{\alpha} = K'\alpha \tag{17}$$

$$\hat{\beta} = \beta\lambda'^{(1/K'\alpha)} \tag{18}$$

where the symbols have the same meaning as for the case of LS parameter estimation.

Properties of Estimated Value of Probability of Crack Exceedance

First, note that for the LS estimation of the probability of crack exceedance, one has

$$\hat{F}_{a(0)} = \exp\left\{-\left[\frac{\ln(x_u/x)}{Q\hat{\beta}}\right]^{\hat{\alpha}}\right\} = \exp -\left\{\frac{Q\beta[-\ln(F_{a(0)})]^{1/\alpha}}{Q\hat{\beta}}\right\}^{\hat{\alpha}} \tag{19}$$

Hence, together with Eqs. (15) and (16), it follows that

$$\hat{F}_{a(0)} = \exp -\left\{[-\ln(F_{a(0)})]^{1/\alpha}\exp[-(\lambda/\alpha)]\right\}^{\hat{\alpha}}$$
$$= \exp -\left\{[-\ln(F_{a(0)})]\exp(-\lambda)\right\}^{K} \tag{20}$$

Table 5 Lower confidence bound of EIFS distribution (γ = 0.95), LS parameter estimation

	N																			
$\bar{P}_{2l}$	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.50	0.234	0.306	0.338	0.357	0.371	0.381	0.390	0.397	0.402	0.408	0.412	0.415	0.418	0.422	0.424	0.427	0.429	0.430	0.432	0.434
0.52	0.253	0.327	0.359	0.378	0.392	0.403	0.411	0.418	0.423	0.429	0.433	0.436	0.439	0.442	0.445	0.447	0.449	0.451	0.453	0.455
0.54	0.273	0.348	0.380	0.400	0.413	0.424	0.433	0.439	0.445	0.450	0.454	0.457	0.460	0.463	0.466	0.468	0.470	0.472	0.474	0.476
0.56	0.293	0.369	0.402	0.421	0.435	0.446	0.454	0.460	0.466	0.471	0.475	0.478	0.481	0.484	0.487	0.489	0.491	0.492	0.494	0.496
0.58	0.313	0.390	0.422	0.442	0.456	0.467	0.475	0.482	0.487	0.492	0.496	0.499	0.502	0.505	0.508	0.510	0.512	0.513	0.515	0.517
0.60	0.333	0.412	0.444	0.464	0.478	0.489	0.496	0.503	0.508	0.513	0.517	0.521	0.523	0.526	0.529	0.531	0.533	0.534	0.536	0.538
0.62	0.353	0.434	0.466	0.486	0.499	0.510	0.518	0.524	0.530	0.534	0.538	0.542	0.544	0.547	0.550	0.552	0.554	0.555	0.557	0.559
0.64	0.373	0.456	0.488	0.507	0.521	0.532	0.539	0.546	0.551	0.556	0.559	0.563	0.565	0.568	0.571	0.573	0.575	0.576	0.578	0.580
0.66	0.394	0.477	0.510	0.529	0.543	0.553	0.561	0.567	0.572	0.577	0.581	0.584	0.586	0.589	0.593	0.594	0.596	0.597	0.599	0.601
0.68	0.415	0.500	0.532	0.551	0.565	0.575	0.582	0.589	0.594	0.599	0.602	0.605	0.608	0.610	0.614	0.616	0.617	0.618	0.620	0.622
0.70	0.436	0.522	0.555	0.574	0.587	0.597	0.604	0.611	0.615	0.621	0.624	0.627	0.629	0.632	0.635	0.637	0.638	0.640	0.641	0.643
0.72	0.458	0.545	0.577	0.596	0.609	0.619	0.626	0.633	0.637	0.642	0.646	0.649	0.651	0.653	0.657	0.658	0.660	0.661	0.662	0.664
0.74	0.480	0.569	0.601	0.619	0.632	0.642	0.649	0.655	0.659	0.664	0.668	0.671	65	70	0.678	0.680	0.681	0.682	0.684	0.685
0.76	0.502	0.593	0.624	0.642	0.655	0.664	0.671	0.678	0.681	0.687	0.690	0.693	0.695	0.697	0.700	0.701	0.703	0.704	0.706	0.707
0.78	0.526	0.617	0.648	0.666	0.678	0.687	0.694	0.700	0.704	0.709	0.712	0.715	0.717	0.719	0.722	0.723	0.725	0.726	0.727	0.729
0.80	0.552	0.642	0.672	0.690	0.702	0.711	0.718	0.723	0.727	0.732	0.735	0.738	0.739	0.742	0.744	0.745	0.747	0.748	0.749	0.751
0.82	0.577	0.668	0.697	0.715	0.726	0.735	0.741	0.747	0.750	0.755	0.757	0.760	0.762	0.764	0.767	0.768	0.769	0.770	0.771	0.773
0.84	0.604	0.694	0.723	0.740	0.751	0.759	0.765	0.771	0.774	0.778	0.781	0.783	0.785	0.787	0.790	0.791	0.792	0.793	0.794	0.796
0.86	0.633	0.722	0.749	0.766	0.776	0.784	0.790	0.795	0.798	0.802	0.805	0.807	0.808	0.810	0.813	0.814	0.815	0.816	0.817	0.819
0.88	0.663	0.750	0.777	0.793	0.802	0.810	0.815	0.820	0.823	0.826	0.829	0.831	0.832	0.834	0.836	0.837	0.838	0.839	0.840	0.842
0.90	0.697	0.780	0.806	0.820	0.829	0.836	0.841	0.845	0.848	0.851	0.853	0.855	0.857	0.859	0.860	0.861	0.862	0.863	0.864	0.865
0.92	0.731	0.812	0.836	0.849	0.857	0.863	0.868	0.872	0.874	0.877	0.879	0.881	0.882	0.884	0.885	0.886	0.887	0.888	0.888	0.890
0.94	0.771	0.847	0.868	0.880	0.887	0.892	0.896	0.899	0.901	0.904	0.905	0.907	0.908	0.909	0.911	0.911	0.912	0.913	0.913	0.914
0.96	0.816	0.886	0.903	0.912	0.918	0.922	0.926	0.928	0.930	0.932	0.933	0.934	0.935	0.936	0.937	0.938	0.938	0.939	0.939	0.940
0.98	0.874	0.930	0.943	0.949	0.953	0.956	0.958	0.960	0.961	0.962	0.963	0.964	0.964	0.965	0.966	0.966	0.966	0.967	0.967	0.968

Table 6 Lower confidence bound of EIFS distribution (γ = 0.98), LS parameter estimation

	N																			
$\bar{P}_{2l}$	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.50	0.180	0.262	0.299	0.322	0.339	0.353	0.363	0.371	0.379	0.385	0.390	0.394	0.398	0.401	0.405	0.408	0.411	0.413	0.415	0.417
0.52	0.196	0.281	0.319	0.342	0.360	0.373	0.384	0.392	0.399	0.406	0.411	0.415	0.419	0.422	0.425	0.429	0.432	0.434	0.436	0.438
0.54	0.212	0.300	0.339	0.362	0.381	0.394	0.404	0.413	0.420	0.427	0.431	0.435	0.439	0.442	0.446	0.449	0.452	0.454	0.456	0.459
0.56	0.228	0.320	0.360	0.383	0.401	0.415	0.425	0.434	0.441	0.448	0.452	0.456	0.460	0.463	0.467	0.470	0.473	0.475	0.477	0.479
0.58	0.245	0.340	0.380	0.404	0.422	0.435	0.446	0.455	0.462	0.468	0.472	0.477	0.481	0.484	0.488	0.491	0.493	0.496	0.497	0.500
0.60	0.262	0.360	0.401	0.425	0.443	0.456	0.467	0.475	0.484	0.489	0.493	0.498	0.502	0.505	0.509	0.511	0.514	0.516	0.518	0.520
0.62	0.281	0.380	0.422	0.446	0.464	0.477	0.488	0.496	0.503	0.510	0.514	0.519	0.522	0.526	0.530	0.532	0.535	0.537	0.539	0.541
0.64	0.298	0.400	0.443	0.466	0.485	0.499	0.509	0.517	0.525	0.531	0.535	0.540	0.543	0.547	0.551	0.553	0.556	0.558	0.560	0.562
0.66	0.317	0.420	0.464	0.488	0.506	0.520	0.530	0.539	0.546	0.552	0.557	0.561	0.564	0.568	0.572	0.574	0.577	0.579	0.581	0.583
0.68	0.336	0.442	0.486	0.510	0.528	0.541	0.552	0.560	0.568	0.574	0.579	0.582	0.586	0.589	0.593	0.595	0.598	0.599	0.602	0.604
0.70	0.355	0.464	0.508	0.532	0.550	0.563	0.573	0.582	0.590	0.595	0.600	0.604	0.607	0.611	0.614	0.616	0.619	0.621	0.623	0.625
0.72	0.375	0.486	0.530	0.554	0.572	0.586	0.595	0.605	0.612	0.617	0.622	0.626	0.629	0.632	0.636	0.638	0.641	0.642	0.645	0.647
0.74	0.396	0.509	0.553	0.578	0.595	0.608	0.618	0.627	0.634	0.639	0.644	0.648	0.651	0.654	0.658	0.660	0.663	0.664	0.666	0.668
0.76	0.417	0.533	0.576	0.601	0.618	0.631	0.641	0.650	0.656	0.662	0.666	0.670	0.673	0.677	0.680	0.682	0.684	0.685	0.688	0.690
0.78	0.440	0.557	0.600	0.625	0.641	0.655	0.664	0.672	0.679	0.685	0.689	0.693	0.695	0.699	0.702	0.704	0.707	0.707	0.710	0.712
0.80	0.463	0.582	0.625	0.650	0.666	0.678	0.688	0.696	0.702	0.707	0.712	0.716	0.718	0.722	0.725	0.727	0.729	0.730	0.733	0.735
0.82	0.487	0.608	0.651	0.675	0.691	0.703	0.712	0.720	0.726	0.731	0.735	0.739	0.742	0.745	0.748	0.750	0.752	0.753	0.755	0.757
0.84	0.513	0.635	0.677	0.701	0.716	0.728	0.737	0.745	0.750	0.755	0.759	0.763	0.765	0.769	0.771	0.773	0.775	0.776	0.778	0.780
0.86	0.540	0.663	0.705	0.728	0.743	0.754	0.763	0.770	0.775	0.780	0.784	0.787	0.790	0.793	0.796	0.797	0.799	0.800	0.802	0.804
0.88	0.569	0.693	0.734	0.756	0.771	0.781	0.790	0.797	0.801	0.806	0.809	0.812	0.815	0.818	0.820	0.822	0.823	0.824	0.826	0.828
0.90	0.602	0.725	0.764	0.786	0.800	0.810	0.818	0.824	0.827	0.832	0.836	0.838	0.840	0.843	0.845	0.847	0.848	0.849	0.851	0.852
0.92	0.637	0.759	0.797	0.817	0.830	0.839	0.846	0.852	0.855	0.859	0.863	0.865	0.867	0.869	0.872	0.873	0.874	0.875	0.876	0.878
0.94	0.677	0.797	0.832	0.851	0.863	0.871	0.877	0.882	0.885	0.888	0.891	0.893	0.895	0.897	0.899	0.900	0.901	0.902	0.903	0.904
0.96	0.728	0.841	0.873	0.888	0.898	0.905	0.910	0.914	0.917	0.920	0.922	0.923	0.925	0.926	0.928	0.929	0.929	0.930	0.931	0.932
0.98	0.796	0.896	0.920	0.932	0.939	0.944	0.947	0.950	0.952	0.954	0.955	0.957	0.958	0.958	0.959	0.960	0.961	0.961	0.962	0.962

In a similar way for ML estimation, it is

$$\hat{F}_{a(0)} = \exp\left\{ -\frac{[-\ln(F_{a(0)})]^{K'}}{\lambda'} \right\} \tag{21}$$

As a consequence, the distribution of $F_{a(0)}$ can be studied through the distribution of its estimated value, $\hat{F}_{a(0)}$, for both ML and LS estimation procedures.

Exact Confidence Limits for Probability of Crack Exceedance

Because the distribution of $F_{a(0)}$ is dependent on the $\hat{F}_{a(0)}$ distribution, a confidence limit for the actual distribution can be determined through the knowledge of its estimate. Monte Carlo simulations have been used to obtain the distribution of the actual probability of crack exceedance for a given value of its estimate. The number of Monte Carlo simulations used for the calculations was 64,000 for all cases examined. Sets of random numbers with uniform distribution have been generated and then, given the α and β values, transformed into random numbers with compatible Weibull distribution. For a fixed value of $\hat{F}_{a(0)}$ a discrete evaluation of the probability distribution function of $F_{a(0)}$ is obtained, and the confidence bound of interest is derived. One is also interested in knowing the lower confidence bound of the probability of having a crack with dimension greater than a given one, which was named $\bar{P}_{1l}$ in one of the preceding paragraphs and which can be expressed by $1 - F_{a(0)}$. Hence, the lower confidence bounds for P_1 can be obtained by

Table 7 Upper confidence bound of EIFS distribution (γ = 0.90), ML parameter estimation

	N																			
$\bar{P}_{2u}$	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.02	0.255	0.135	0.097	0.079	0.069	0.062	0.058	0.054	0.051	0.048	0.046	0.044	0.043	0.042	0.041	0.040	0.039	0.038	0.038	0.037
0.04	0.298	0.177	0.137	0.116	0.105	0.097	0.091	0.086	0.082	0.079	0.076	0.074	0.073	0.071	0.070	0.068	0.067	0.066	0.065	0.064
0.06	0.330	0.210	0.168	0.147	0.135	0.126	0.119	0.114	0.110	0.106	0.103	0.101	0.099	0.097	0.096	0.094	0.093	0.091	0.091	0.089
0.08	0.357	0.239	0.197	0.174	0.162	0.152	0.146	0.140	0.135	0.131	0.128	0.126	0.124	0.122	0.120	0.119	0.117	0.116	0.114	0.113
0.10	0.380	0.265	0.222	0.200	0.188	0.177	0.170	0.165	0.160	0.156	0.152	0.150	0.147	0.145	0.144	0.142	0.140	0.139	0.138	0.136
0.12	0.402	0.289	0.247	0.224	0.212	0.201	0.195	0.188	0.183	0.179	0.175	0.173	0.170	0.168	0.166	0.165	0.163	0.161	0.160	0.159
0.14	0.422	0.312	0.271	0.248	0.235	0.224	0.218	0.211	0.206	0.202	0.198	0.195	0.193	0.191	0.189	0.187	0.185	0.183	0.182	0.181
0.16	0.441	0.334	0.294	0.271	0.257	0.247	0.240	0.234	0.228	0.224	0.220	0.217	0.215	0.213	0.211	0.209	0.207	0.205	0.204	0.203
0.18	0.459	0.356	0.316	0.293	0.279	0.269	0.262	0.256	0.250	0.246	0.242	0.239	0.237	0.234	0.232	0.231	0.228	0.227	0.225	0.224
0.20	0.477	0.376	0.337	0.314	0.301	0.291	0.284	0.277	0.272	0.267	0.263	0.261	0.258	0.256	0.254	0.252	0.250	0.248	0.247	0.245
0.22	0.494	0.397	0.358	0.336	0.322	0.312	0.305	0.299	0.293	0.289	0.285	0.282	0.280	0.277	0.275	0.273	0.271	0.269	0.268	0.266
0.24	0.510	0.417	0.379	0.357	0.343	0.333	0.326	0.320	0.315	0.310	0.306	0.303	0.301	0.298	0.296	0.294	0.292	0.290	0.289	0.287
0.26	0.527	0.437	0.400	0.378	0.364	0.355	0.347	0.341	0.335	0.331	0.327	0.324	0.322	0.319	0.317	0.315	0.313	0.311	0.310	0.308
0.28	0.544	0.456	0.420	0.398	0.385	0.375	0.367	0.361	0.356	0.351	0.347	0.345	0.342	0.339	0.338	0.336	0.333	0.332	0.330	0.329
0.30	0.559	0.475	0.440	0.418	0.405	0.396	0.388	0.382	0.377	0.372	0.368	0.365	0.363	0.360	0.358	0.356	0.354	0.352	0.351	0.350
0.32	0.575	0.494	0.460	0.439	0.425	0.416	0.409	0.402	0.397	0.392	0.389	0.386	0.383	0.381	0.379	0.377	0.374	0.373	0.372	0.370
0.34	0.591	0.513	0.479	0.459	0.445	0.436	0.429	0.423	0.418	0.413	0.409	0.406	0.404	0.401	0.399	0.398	0.395	0.393	0.392	0.390
0.36	0.607	0.532	0.499	0.479	0.465	0.456	0.449	0.443	0.438	0.433	0.429	0.427	0.424	0.421	0.419	0.418	0.415	0.414	0.412	0.411
0.38	0.622	0.550	0.518	0.498	0.485	0.476	0.469	0.463	0.458	0.453	0.450	0.447	0.444	0.442	0.439	0.438	0.436	0.434	0.433	0.431
0.40	0.637	0.568	0.537	0.518	0.505	0.496	0.489	0.483	0.478	0.473	0.470	0.467	0.460	0.462	0.460	0.458	0.456	0.454	0.453	0.451
0.42	0.653	0.586	0.555	0.537	0.524	0.516	0.508	0.503	0.498	0.493	0.490	0.487	0.485	0.482	0.480	0.478	0.476	0.474	0.473	0.471
0.44	0.668	0.604	0.574	0.556	0.544	0.535	0.528	0.522	0.518	0.513	0.510	0.507	0.505	0.502	0.500	0.498	0.496	0.494	0.493	0.492
0.46	0.683	0.622	0.593	0.575	0.563	0.554	0.548	0.542	0.538	0.533	0.529	0.527	0.525	0.522	0.519	0.518	0.516	0.514	0.513	0.512
0.48	0.698	0.639	0.612	0.594	0.583	0.574	0.567	0.561	0.557	0.552	0.549	0.546	0.544	0.542	0.539	0.538	0.536	0.534	0.533	0.531
0.50	0.713	0.656	0.630	0.613	0.601	0.593	0.586	0.581	0.577	0.572	0.569	0.566	0.564	0.561	0.559	0.558	0.555	0.554	0.553	0.551

Table 8 Upper confidence bound of EIFS distribution (γ = 0.95), ML parameter estimation

	N																			
$\bar{P}_{2u}$	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.02	0.340	0.181	0.129	0.102	0.088	0.079	0.072	0.066	0.062	0.059	0.056	0.053	0.052	0.050	0.048	0.047	0.046	0.045	0.044	0.043
0.04	0.382	0.227	0.172	0.143	0.127	0.116	0.108	0.102	0.096	0.092	0.088	0.086	0.084	0.081	0.079	0.078	0.076	0.075	0.074	0.072
0.06	0.412	0.261	0.206	0.176	0.159	0.147	0.139	0.132	0.126	0.121	0.117	0.114	0.112	0.109	0.107	0.105	0.103	0.102	0.100	0.099
0.08	0.437	0.291	0.235	0.205	0.188	0.176	0.166	0.159	0.152	0.148	0.143	0.140	0.138	0.135	0.133	0.130	0.128	0.127	0.125	0.124
0.10	0.458	0.317	0.263	0.231	0.214	0.202	0.192	0.184	0.178	0.173	0.168	0.165	0.162	0.159	0.157	0.155	0.152	0.151	0.149	0.148
0.12	0.478	0.341	0.287	0.256	0.239	0.226	0.216	0.208	0.202	0.197	0.192	0.189	0.186	0.183	0.181	0.178	0.176	0.174	0.173	0.171
0.14	0.498	0.364	0.311	0.280	0.263	0.250	0.240	0.232	0.226	0.220	0.215	0.212	0.209	0.206	0.203	0.201	0.198	0.197	0.195	0.193
0.16	0.516	0.386	0.334	0.304	0.286	0.273	0.263	0.255	0.248	0.243	0.238	0.235	0.231	0.228	0.226	0.223	0.221	0.219	0.217	0.215
0.18	0.533	0.407	0.356	0.326	0.309	0.295	0.285	0.278	0.271	0.265	0.260	0.257	0.253	0.250	0.248	0.245	0.243	0.241	0.239	0.237
0.20	0.550	0.427	0.377	0.348	0.330	0.317	0.307	0.300	0.293	0.287	0.282	0.279	0.275	0.272	0.269	0.267	0.264	0.262	0.261	0.259
0.22	0.567	0.447	0.398	0.370	0.352	0.339	0.329	0.321	0.314	0.309	0.304	0.301	0.297	0.293	0.291	0.289	0.286	0.284	0.282	0.280
0.24	0.582	0.467	0.419	0.391	0.373	0.360	0.350	0.343	0.336	0.330	0.325	0.322	0.318	0.315	0.312	0.310	0.307	0.305	0.303	0.301
0.26	0.597	0.486	0.439	0.411	0.394	0.381	0.371	0.364	0.357	0.351	0.346	0.343	0.339	0.336	0.333	0.331	0.328	0.326	0.324	0.322
0.28	0.612	0.505	0.459	0.432	0.415	0.402	0.392	0.385	0.378	0.372	0.367	0.364	0.360	0.357	0.354	0.352	0.349	0.347	0.345	0.343
0.30	0.627	0.524	0.479	0.452	0.435	0.423	0.413	0.406	0.398	0.393	0.388	0.384	0.381	0.378	0.375	0.373	0.370	0.368	0.366	0.364
0.32	0.641	0.542	0.499	0.472	0.455	0.443	0.434	0.426	0.419	0.413	0.408	0.405	0.401	0.398	0.395	0.394	0.390	0.388	0.386	0.385
0.34	0.656	0.560	0.518	0.492	0.475	0.463	0.454	0.447	0.439	0.433	0.429	0.425	0.422	0.419	0.416	0.414	0.411	0.409	0.407	0.405
0.36	0.670	0.578	0.536	0.511	0.495	0.484	0.474	0.467	0.460	0.454	0.449	0.446	0.442	0.439	0.437	0.434	0.431	0.429	0.427	0.426
0.38	0.684	0.596	0.556	0.530	0.515	0.504	0.494	0.487	0.480	0.474	0.469	0.466	0.462	0.459	0.457	0.455	0.452	0.450	0.447	0.446
0.40	0.698	0.613	0.574	0.550	0.534	0.523	0.514	0.507	0.500	0.494	0.489	0.487	0.483	0.479	0.477	0.475	0.472	0.470	0.468	0.466
0.42	0.712	0.630	0.592	0.569	0.554	0.543	0.534	0.527	0.520	0.514	0.510	0.507	0.503	0.500	0.497	0.495	0.492	0.490	0.488	0.486
0.44	0.726	0.648	0.611	0.588	0.573	0.562	0.553	0.546	0.540	0.534	0.530	0.527	0.523	0.520	0.517	0.515	0.512	0.510	0.508	0.506
0.46	0.740	0.665	0.629	0.606	0.592	0.581	0.573	0.566	0.559	0.554	0.549	0.546	0.543	0.539	0.537	0.535	0.532	0.530	0.528	0.526
0.48	0.753	0.682	0.648	0.625	0.611	0.601	0.592	0.585	0.579	0.573	0.569	0.566	0.562	0.559	0.557	0.555	0.552	0.550	0.548	0.546
0.50	0.766	0.698	0.666	0.644	0.630	0.619	0.611	0.604	0.598	0.593	0.589	0.585	0.582	0.579	0.576	0.574	0.572	0.570	0.568	0.566

taking the complement to one of the upper confidence bounds of $F_{a(0)}$.

The lower confidence bounds of the compatible Weibull distribution function $\bar{P}_{2l}$ are reported in Tables 1, 2, and 3 for ML parameter estimation corresponding to $\gamma = 0.90$, 0.95, and 0.98, respectively, varying the data sample amplitude N. The same data for LS parameter estimation are reported in Tables 4, 5, and 6 for the same γ values and data sample amplitude N. The upper confidence bounds, $\bar{P}_{2u}$, used to obtain the lower confidence bounds for the probability of having a crack with the dimension greater than a fixed one, $\bar{P}_{1l}$, are reported in Tables 7, 8, and 9 for ML parameter estimation for $\gamma = 0.90$, 0.95, and 0.98, respectively, and in Tables 10, 11, and 12 for LS parameter estimation for the same confidence levels.

As for the lower confidence bound, the results are reported as a function of the data sample amplitude N. The data reported in Tables 1–3 were already reported elsewhere,[12] in a less extensive way, as lower confidence bounds of the Weibull reliability function. In fact, the Weibull reliability function has the same expression as the compatible Weibull distribution function [see Eq. (3)]. Some small differences (some units in the third decimal digit of the value reported in Tables 1–3) between the results reported here and the ones reported by Thoman et al.[12] are abscribed to the different number of executed simulations—64,000 here vs 10,000 by Thoman et al.[12]

More extensive results for all the cases reported in terms of sample data amplitude and probability levels can be found in Pieracci.[13]

Table 9 Upper confidence bound of EIFS distribution (γ = 0.98), ML parameter estimation

	N																			
$\bar{P}_{2u}$	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.02	0.439	0.241	0.170	0.134	0.115	0.100	0.090	0.083	0.077	0.072	0.068	0.065	0.062	0.059	0.058	0.056	0.054	0.053	0.052	0.050
0.04	0.479	0.289	0.216	0.178	0.157	0.141	0.130	0.121	0.114	0.109	0.104	0.101	0.097	0.094	0.092	0.089	0.087	0.085	0.084	0.082
0.06	0.507	0.325	0.251	0.213	0.190	0.174	0.162	0.152	0.146	0.139	0.134	0.131	0.126	0.123	0.121	0.118	0.115	0.114	0.112	0.110
0.08	0.530	0.354	0.281	0.243	0.220	0.204	0.191	0.181	0.174	0.167	0.162	0.158	0.153	0.150	0.148	0.145	0.142	0.140	0.138	0.136
0.10	0.550	0.380	0.308	0.271	0.247	0.231	0.218	0.208	0.200	0.193	0.188	0.183	0.179	0.175	0.173	0.170	0.167	0.165	0.163	0.160
0.12	0.566	0.404	0.334	0.296	0.272	0.256	0.243	0.233	0.225	0.217	0.212	0.208	0.203	0.200	0.197	0.194	0.191	0.189	0.187	0.184
0.14	0.582	0.427	0.357	0.320	0.297	0.281	0.267	0.257	0.249	0.241	0.236	0.232	0.227	0.223	0.221	0.217	0.214	0.212	0.210	0.207
0.16	0.599	0.448	0.380	0.343	0.320	0.304	0.291	0.280	0.272	0.264	0.259	0.255	0.250	0.246	0.243	0.240	0.237	0.235	0.233	0.230
0.18	0.615	0.469	0.402	0.365	0.343	0.327	0.313	0.303	0.295	0.287	0.282	0.278	0.273	0.269	0.265	0.263	0.259	0.257	0.255	0.252
0.20	0.630	0.488	0.423	0.387	0.365	0.349	0.336	0.325	0.317	0.310	0.305	0.300	0.295	0.291	0.287	0.285	0.281	0.279	0.277	0.274
0.22	0.644	0.507	0.444	0.408	0.387	0.371	0.358	0.347	0.339	0.332	0.326	0.322	0.316	0.313	0.309	0.307	0.303	0.301	0.298	0.296
0.24	0.657	0.525	0.464	0.429	0.408	0.392	0.379	0.368	0.361	0.353	0.348	0.344	0.338	0.334	0.331	0.328	0.324	0.322	0.320	0.317
0.26	0.671	0.543	0.484	0.449	0.429	0.413	0.400	0.390	0.382	0.375	0.369	0.365	0.359	0.356	0.352	0.349	0.345	0.343	0.341	0.339
0.28	0.685	0.560	0.504	0.470	0.450	0.434	0.421	0.410	0.403	0.395	0.390	0.386	0.380	0.377	0.373	0.370	0.366	0.364	0.362	0.359
0.30	0.699	0.577	0.523	0.490	0.470	0.454	0.442	0.431	0.424	0.416	0.411	0.407	0.401	0.398	0.394	0.391	0.388	0.385	0.383	0.380
0.32	0.711	0.594	0.542	0.509	0.490	0.474	0.462	0.452	0.445	0.437	0.432	0.427	0.422	0.418	0.415	0.412	0.409	0.406	0.404	0.401
0.34	0.723	0.611	0.562	0.529	0.510	0.494	0.482	0.473	0.466	0.458	0.452	0.448	0.442	0.439	0.435	0.432	0.429	0.426	0.424	0.422
0.36	0.736	0.628	0.581	0.548	0.530	0.514	0.502	0.493	0.486	0.478	0.473	0.468	0.463	0.459	0.456	0.453	0.450	0.447	0.445	0.442
0.38	0.748	0.646	0.599	0.567	0.549	0.534	0.522	0.513	0.506	0.498	0.493	0.488	0.483	0.479	0.476	0.473	0.471	0.467	0.465	0.463
0.40	0.760	0.662	0.617	0.586	0.568	0.553	0.542	0.533	0.526	0.518	0.512	0.508	0.503	0.499	0.496	0.593	0.491	0.487	0.485	0.483
0.42	0.772	0.679	0.635	0.605	0.587	0.572	0.561	0.552	0.545	0.538	0.532	0.528	0.523	0.519	0.516	0.514	0.511	0.507	0.505	0.503
0.44	0.785	0.695	0.653	0.623	0.606	0.592	0.581	0.572	0.565	0.557	0.552	0.548	0.543	0.540	0.536	0.534	0.531	0.527	0.525	0.523
0.46	0.797	0.711	0.671	0.642	0.624	0.611	0.601	0.592	0.584	0.577	0.572	0.568	0.563	0.560	0.556	0.553	0.550	0.547	0.545	0.543
0.48	0.808	0.727	0.688	0.660	0.643	0.630	0.620	0.611	0.604	0.596	0.591	0.587	0.582	0.579	0.575	0.573	0.570	0.567	0.565	0.563
0.50	0.820	0.743	0.704	0.677	0.661	0.648	0.638	0.630	0.623	0.615	0.611	0.607	0.602	0.599	0.595	0.593	0.590	0.586	0.585	0.583

Table 10 Upper confidence bound of EIFS distribution (γ = 0.90), LS parameter estimation

	N																			
$\bar{P}_{2u}$	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.02	0.102	0.058	0.048	0.043	0.041	0.039	0.038	0.036	0.036	0.035	0.034	0.033	0.033	0.033	0.032	0.032	0.031	0.031	0.031	0.031
0.04	0.143	0.092	0.080	0.074	0.070	0.068	0.066	0.064	0.063	0.062	0.061	0.060	0.059	0.059	0.058	0.058	0.057	0.057	0.056	0.056
0.06	0.176	0.122	0.108	0.101	0.097	0.094	0.092	0.090	0.088	0.087	0.085	0.084	0.084	0.083	0.082	0.082	0.081	0.081	0.080	0.080
0.08	0.206	0.149	0.135	0.127	0.122	0.119	0.117	0.114	0.113	0.110	0.109	0.108	0.107	0.106	0.105	0.105	0.104	0.104	0.103	0.103
0.10	0.235	0.176	0.161	0.152	0.147	0.143	0.141	0.138	0.136	0.134	0.132	0.131	0.130	0.129	0.128	0.128	0.127	0.126	0.126	0.125
0.12	0.261	0.202	0.186	0.176	0.171	0.167	0.164	0.161	0.159	0.157	0.155	0.154	0.153	0.152	0.151	0.150	0.149	0.148	0.148	0.147
0.14	0.287	0.228	0.210	0.200	0.195	0.190	0.187	0.184	0.182	0.180	0.178	0.176	0.175	0.174	0.173	0.173	0.171	0.170	0.170	0.169
0.16	0.312	0.253	0.235	0.224	0.218	0.213	0.210	0.206	0.204	0.202	0.200	0.198	0.197	0.196	0.195	0.194	0.193	0.192	0.191	0.191
0.18	0.338	0.278	0.259	0.248	0.242	0.236	0.233	0.229	0.227	0.224	0.222	0.220	0.219	0.218	0.217	0.216	0.215	0.214	0.213	0.212
0.20	0.362	0.303	0.284	0.271	0.264	0.259	0.255	0.251	0.249	0.246	0.244	0.242	0.241	0.240	0.239	0.238	0.236	0.235	0.234	0.234
0.22	0.387	0.327	0.308	0.294	0.287	0.282	0.278	0.274	0.271	0.268	0.266	0.264	0.263	0.261	0.260	0.259	0.258	0.257	0.256	0.255
0.24	0.411	0.352	0.331	0.318	0.310	0.305	0.300	0.296	0.293	0.290	0.288	0.286	0.285	0.283	0.282	0.281	0.279	0.278	0.277	0.276
0.26	0.436	0.375	0.355	0.341	0.333	0.328	0.323	0.318	0.315	0.312	0.310	0.308	0.306	0.304	0.303	0.302	0.301	0.299	0.298	0.298
0.28	0.460	0.400	0.378	0.364	0.356	0.350	0.345	0.341	0.337	0.334	0.332	0.330	0.328	0.326	0.325	0.324	0.322	0.320	0.320	0.319
0.30	0.485	0.425	0.402	0.387	0.378	0.373	0.367	0.363	0.360	0.356	0.353	0.352	0.350	0.348	0.346	0.345	0.344	0.342	0.341	0.340
0.32	0.509	0.449	0.426	0.411	0.401	0.395	0.390	0.385	0.382	0.378	0.375	0.373	0.371	0.369	0.368	0.367	0.365	0.363	0.362	0.361
0.34	0.532	0.474	0.450	0.434	0.424	0.418	0.412	0.407	0.404	0.400	0.397	0.395	0.393	0.391	0.389	0.388	0.386	0.385	0.384	0.383
0.36	0.558	0.498	0.473	0.458	0.447	0.441	0.435	0.430	0.426	0.422	0.419	0.417	0.415	0.413	0.411	0.410	0.408	0.406	0.405	0.404
0.38	0.582	0.523	0.497	0.781	0.470	0.463	0.457	0.452	0.448	0.444	0.441	0.438	0.436	0.434	0.432	0.431	0.429	0.428	0.426	0.425
0.40	0.607	0.548	0.521	0.504	0.493	0.486	0.480	0.474	0.470	0.466	0.463	0.461	0.458	0.456	0.454	0.453	0.451	0.449	0.448	0.446
0.42	0.632	0.572	0.545	0.528	0.516	0.508	0.502	0.496	0.492	0.487	0.484	0.482	0.480	0.477	0.475	0.474	0.472	0.470	0.469	0.468
0.44	0.657	0.597	0.568	0.551	0.539	0.531	0.524	0.518	0.514	0.510	0.506	0.504	0.502	0.499	0.497	0.495	0.493	0.492	0.490	0.489
0.46	0.682	0.620	0.592	0.574	0.562	0.553	0.546	0.541	0.536	0.532	0.528	0.525	0.523	0.520	0.518	0.517	0.515	0.513	0.512	0.511
0.48	0.706	0.645	0.615	0.597	0.585	0.576	0.568	0.563	0.558	0.553	0.550	0.547	0.545	0.542	0.540	0.538	0.536	0.535	0.533	0.532
0.50	0.730	0.668	0.639	0.620	0.607	0.598	0.590	0.585	0.580	0.575	0.572	0.568	0.566	0.563	0.561	0.560	0.557	0.556	0.554	0.553

Example

An example is reported to clarify the described procedure. Let us consider a wing panel whose fatigue quality may be estimated through the following TTCI data sample: 4511, 6875, 7474, 9905, 10,262, 12,818, 14,066, 16,676, 18,653, and 23,536, which represent the time, in terms of flight hours, that is necessary to reach a crack length of 0.889 mm (0.035 in.) under the F-16 400-h loading history for a 15% load transfer specimen set (the set that was named AFXLR4 by Manning and Yang[8]). The estimated values of the parameters are $\hat{\alpha} = 1.575$ and $\hat{\beta} = 12{,}388$ for the case of LS estimation; also, it is assumed that x_u is known and equal to 0.635 mm (0.025 in.) and that the values of the parameters of the crack propagation law reported in Eq. (5) are $Q = 1.822E - 4$ and $b = 1$; these values are obtained in the example problem discussed by Manning and Yang[8] for showing the derivation of the parameters of the USAF method. It follows also from Eqs. (7) and (8) that $\hat{\phi} = 2.257$ and $\varepsilon = 1847$. Suppose now that one is interested in obtaining the maintenance interval such that if the component is inspected in such an interval, there is a 0.95 probability that the probability of having a crack longer than 0.762 mm (0.03 in.) is 0.5 and a 0.95 probability that the probability of having a crack with length smaller than 2.286 mm (0.09 in.) is 0.9. In the terminology used so far, it means that $\gamma = 0.95$, $P_1 = 0.5$, $P_2 = 0.9$, $a_1 = 0.762$ mm, and $a_2 = 2.286$ mm. Using Table 5 for the case of a data set of 10, it follows that, for $\gamma = 0.95$, $\bar{P}_{2l} = 0.966$, whereas using Table 11 it follows that $\bar{P}_{1l} = 1 - \bar{P}_{2u} = 0.68$. The corresponding values

Table 11 Upper confidence bound of EIFS distribution (γ = 0.95), LS parameter estimation

	N																			
$\bar{P}_{2u}$	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.02	0.178	0.092	0.072	0.063	0.058	0.053	0.050	0.048	0.046	0.044	0.043	0.042	0.042	0.041	0.040	0.039	0.039	0.038	0.038	0.037
0.04	0.224	0.133	0.110	0.098	0.092	0.086	0.083	0.079	0.077	0.075	0.073	0.072	0.071	0.069	0.069	0.068	0.067	0.066	0.065	0.065
0.06	0.260	0.168	0.142	0.128	0.121	0.115	0.111	0.107	0.105	0.102	0.100	0.098	0.097	0.095	0.095	0.094	0.093	0.092	0.091	0.090
0.08	0.292	0.199	0.172	0.157	0.149	0.142	0.138	0.134	0.130	0.127	0.125	0.124	0.122	0.120	0.119	0.118	0.117	0.116	0.115	0.114
0.10	0.320	0.227	0.199	0.184	0.175	0.168	0.163	0.159	0.155	0.151	0.150	0.148	0.146	0.144	0.143	0.142	0.140	0.139	0.138	0.137
0.12	0.346	0.254	0.226	0.209	0.200	0.193	0.188	0.183	0.179	0.175	0.173	0.171	0.169	0.167	0.166	0.165	0.163	0.162	0.161	0.160
0.14	0.371	0.280	0.251	0.234	0.225	0.217	0.211	0.207	0.202	0.199	0.196	0.194	0.192	0.190	0.189	0.188	0.185	0.184	0.184	0.182
0.16	0.396	0.306	0.276	0.258	0.248	0.240	0.235	0.230	0.225	0.222	0.219	0.217	0.215	0.213	0.211	0.210	0.208	0.207	0.206	0.205
0.18	0.420	0.331	0.300	0.283	0.272	0.264	0.258	0.253	0.249	0.245	0.242	0.240	0.237	0.235	0.234	0.232	0.230	0.228	0.227	0.226
0.20	0.443	0.355	0.324	0.306	0.295	0.287	0.281	0.276	0.271	0.267	0.264	0.262	0.259	0.257	0.255	0.254	0.251	0.250	0.249	0.248
0.22	0.465	0.380	0.348	0.329	0.319	0.310	0.303	0.298	0.294	0.289	0.286	0.284	0.281	0.279	0.277	0.276	0.273	0.272	0.271	0.270
0.24	0.488	0.404	0.372	0.353	0.341	0.333	0.325	0.321	0.316	0.312	0.308	0.306	0.303	0.301	0.299	0.298	0.295	0.293	0.292	0.291
0.26	0.512	0.428	0.396	0.377	0.364	0.355	0.348	0.343	0.338	0.334	0.330	0.328	0.325	0.322	0.321	0.319	0.317	0.315	0.314	0.313
0.28	0.534	0.453	0.419	0.400	0.387	0.378	0.370	0.365	0.360	0.356	0.352	0.349	0.346	0.344	0.342	0.341	0.338	0.336	0.335	0.334
0.30	0.557	0.476	0.443	0.423	0.410	0.400	0.393	0.388	0.382	0.378	0.374	0.371	0.368	0.366	0.364	0.362	0.359	0.358	0.357	0.356
0.32	0.580	0.500	0.466	0.446	0.432	0.423	0.415	0.410	0.404	0.399	0.396	0.393	0.390	0.387	0.385	0.384	0.381	0.379	0.378	0.377
0.34	0.602	0.523	0.489	0.469	0.455	0.446	0.437	0.432	0.426	0.421	0.418	0.415	0.412	0.409	0.407	0.405	0.402	0.401	0.399	0.398
0.36	0.625	0.546	0.512	0.492	0.478	0.468	0.460	0.454	0.448	0.443	0.440	0.437	0.433	0.430	0.428	0.427	0.424	0.422	0.421	0.420
0.38	0.648	0.570	0.536	0.515	0.500	0.491	0.482	0.477	0.470	0.465	0.461	0.458	0.455	0.452	0.450	0.448	0.445	0.444	0.442	0.441
0.40	0.670	0.593	0.559	0.537	0.523	0.513	0.505	0.498	0.492	0.487	0.483	0.480	0.476	0.474	0.471	0.470	0.467	0.465	0.463	0.462
0.42	0.693	0.617	0.583	0.560	0.546	0.536	0.527	0.520	0.514	0.509	0.505	0.502	0.498	0.495	0.493	0.491	0.488	0.486	0.484	0.483
0.44	0.715	0.641	0.605	0.583	0.569	0.558	0.549	0.542	0.536	0.530	0.526	0.523	0.520	0.517	0.514	0.512	0.510	0.508	0.505	0.504
0.46	0.739	0.664	0.629	0.605	0.591	0.580	0.572	0.564	0.558	0.552	0.548	0.545	0.541	0.538	0.536	0.534	0.531	0.529	0.527	0.526
0.48	0.761	0.687	0.652	0.628	0.613	0.602	0.594	0.586	0.580	0.574	0.570	0.567	0.563	0.560	0.557	0.555	0.552	0.550	0.548	0.547
0.50	0.782	0.710	0.674	0.650	0.636	0.624	0.616	0.608	0.602	0.596	0.591	0.588	0.584	0.581	0.578	0.576	0.574	0.572	0.569	0.567

Table 12 Upper confidence bound of EIFS distribution (γ = 0.98), LS parameter estimation

	N																			
$\bar{P}_{2u}$	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
0.02	0.280	0.146	0.109	0.090	0.080	0.073	0.067	0.064	0.060	0.058	0.056	0.054	0.052	0.051	0.050	0.049	0.048	0.047	0.046	0.045
0.04	0.327	0.192	0.152	0.130	0.119	0.110	0.103	0.100	0.095	0.092	0.089	0.087	0.085	0.082	0.082	0.080	0.079	0.078	0.077	0.075
0.06	0.363	0.229	0.187	0.164	0.152	0.142	0.134	0.130	0.124	0.121	0.118	0.116	0.113	0.111	0.110	0.107	0.106	0.105	0.104	0.102
0.08	0.393	0.260	0.218	0.193	0.181	0.171	0.162	0.158	0.152	0.148	0.145	0.143	0.139	0.136	0.136	0.133	0.131	0.130	0.129	0.127
0.10	0.420	0.290	0.246	0.221	0.209	0.198	0.188	0.184	0.178	0.174	0.170	0.168	0.164	0.161	0.160	0.157	0.156	0.155	0.153	0.152
0.12	0.445	0.318	0.272	0.247	0.235	0.223	0.214	0.209	0.203	0.198	0.194	0.193	0.188	0.185	0.184	0.181	0.180	0.178	0.177	0.175
0.14	0.469	0.345	0.298	0.273	0.260	0.248	0.238	0.233	0.227	0.222	0.218	0.216	0.212	0.208	0.207	0.205	0.203	0.201	0.200	0.198
0.16	0.492	0.369	0.324	0.299	0.285	0.272	0.263	0.257	0.250	0.245	0.242	0.239	0.235	0.231	0.230	0.228	0.225	0.224	0.222	0.220
0.18	0.514	0.394	0.349	0.322	0.309	0.296	0.286	0.280	0.273	0.269	0.264	0.262	0.257	0.254	0.252	0.250	0.248	0.246	0.244	0.242
0.20	0.535	0.418	0.372	0.346	0.332	0.319	0.310	0.303	0.296	0.291	0.287	0.284	0.280	0.277	0.274	0.272	0.270	0.269	0.266	0.264
0.22	0.555	0.442	0.396	0.370	0.356	0.342	0.333	0.326	0.319	0.314	0.309	0.306	0.302	0.298	0.296	0.294	0.291	0.291	0.288	0.286
0.24	0.576	0.464	0.419	0.393	0.378	0.365	0.355	0.349	0.341	0.336	0.332	0.329	0.324	0.321	0.318	0.316	0.313	0.312	0.310	0.307
0.26	0.595	0.486	0.442	0.416	0.401	0.388	0.378	0.371	0.364	0.358	0.354	0.351	0.346	0.343	0.340	0.338	0.335	0.334	0.331	0.329
0.28	0.615	0.509	0.466	0.439	0.424	0.411	0.400	0.393	0.386	0.380	0.376	0.373	0.368	0.365	0.362	0.359	0.357	0.355	0.353	0.350
0.30	0.636	0.532	0.489	0.462	0.447	0.433	0.423	0.415	0.409	0.402	0.398	0.394	0.390	0.386	0.383	0.381	0.378	0.376	0.374	0.372
0.32	0.657	0.554	0.512	0.484	0.469	0.456	0.445	0.437	0.431	0.424	0.419	0.416	0.412	0.408	0.405	0.403	0.400	0.398	0.396	0.393
0.34	0.677	0.577	0.536	0.508	0.491	0.477	0.467	0.459	0.453	0.446	0.441	0.438	0.433	0.429	0.426	0.424	0.421	0.419	0.417	0.415
0.36	0.697	0.600	0.558	0.530	0.514	0.500	0.489	0.481	0.475	0.468	0.463	0.459	0.455	0.451	0.448	0.446	0.443	0.440	0.438	0.436
0.38	0.718	0.622	0.581	0.553	0.536	0.521	0.512	0.503	0.497	0.490	0.485	0.481	0.476	0.472	0.470	0.467	0.464	0.461	0.459	0.457
0.40	0.737	0.644	0.603	0.575	0.558	0.544	0.534	0.525	0.519	0.511	0.506	0.502	0.498	0.494	0.491	0.488	0.485	0.482	0.481	0.479
0.42	0.756	0.667	0.625	0.597	0.580	0.565	0.555	0.547	0.540	0.533	0.528	0.524	0.519	0.515	0.512	0.510	0.507	0.504	0.502	0.500
0.44	0.777	0.689	0.647	0.619	0.602	0.588	0.578	0.569	0.562	0.554	0.549	0.545	0.541	0.537	0.533	0.531	0.528	0.525	0.523	0.521
0.46	0.796	0.710	0.670	0.641	0.624	0.610	0.599	0.590	0.583	0.576	0.571	0.567	0.562	0.558	0.555	0.552	0.550	0.546	0.544	0.542
0.48	0.815	0.733	0.691	0.663	0.645	0.631	0.621	0.612	0.605	0.597	0.592	0.588	0.583	0.580	0.576	0.573	0.571	0.567	0.565	0.563
0.50	0.833	0.754	0.712	0.684	0.667	0.653	0.642	0.634	0.626	0.618	0.613	0.609	0.604	0.601	0.597	0.595	0.592	0.588	0.586	0.584

of EIFS, obtained by Eq. (6) with the above-reported estimates of the parameters are $\bar{a}_{P_1} = 0.1854$ mm and $\bar{a}_{P_2} = 0.4928$ mm. From Eqs. (13) and (14), it follows that $\bar{t}_1 = 7757$ and $\bar{t}_2 = 8422$ flight hours. Consequently, if one inspects and upgrades the structure for a service time between 7757 and 8422 flight hours, there is 0.95 probability that the probability that the cracks have a length greater than 0.762 mm (0.03 in.) is 0.5 and that the cracks have length lower than 2.286 mm (0.09 in.) is 0.95.

Conclusions

A procedure is proposed to identify upper and lower bounds of the inspection interval for a metallic structure, based on the durability probabilistic model of the USAF. The proposed methodology uses confidence bounds of the probability of crack exceedance to help in identifying the inspection interval. Consequently, the inspection will take place when the structure is damaged to a certain but limited extent, so that the repairing of the structure is economically advantageous. Tables obtained through Monte Carlo simulations are reported for the actual determination of the time interval in which the inspection should take place. Some properties of the estimated parameters are reported, which lead to a general applicability of the numerical tables presented. The proposed procedure is demonstrated through an example calculation.

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